# Two-period DID Identification Illustration

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- $Y_t$ outcome, $D$ binary treatment, $t \in \{0,1\}$ the two periods
- Potential outcomes: $\{Y_t(d) \mid (t,d) \in \{0,1\}^2\}$
- Observed outcome $Y_t = DY_t(1) + (1-D)Y_t(0)$
- Estimand: ATT, $\tau := E[Y_1(1) - Y_1(0) \mid D = 1]$
- Goal: show identification of $\tau$ based on identifying assumptions:
  - **Consistency**: $Y_t = DY_t(1) + (1-D)Y_t(0)$
  - **No Anticipation (NA)**: $E[Y_0(0) \mid D = 1] = E[Y_0(1) \mid D = 1]$
  - **Parallel Trends (PT)**:

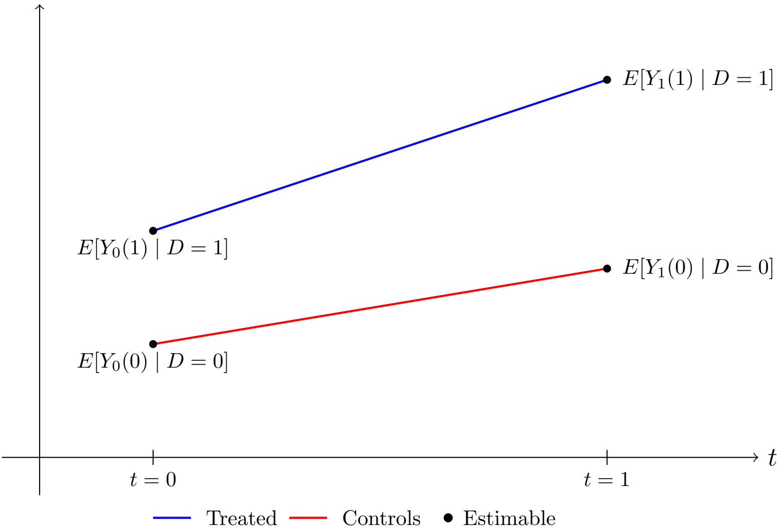
$$E[Y_1(0) - Y_0(0) \mid D = 1] = E[Y_1(0) - Y_0(0) \mid D = 0]$$

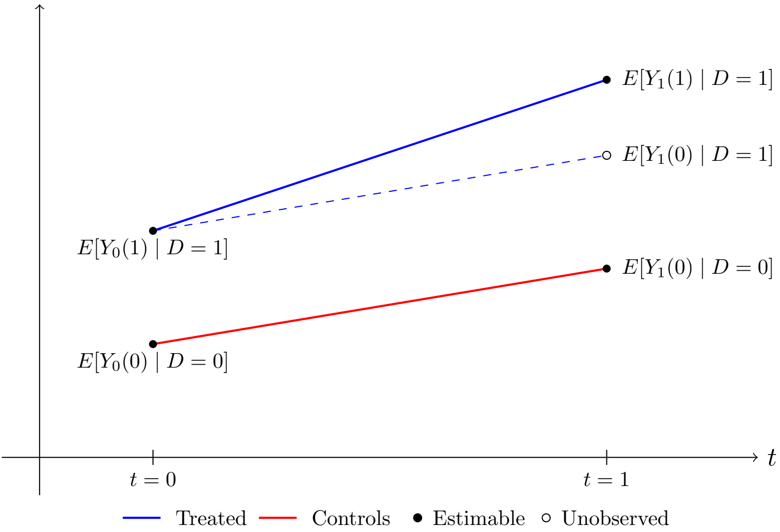
- **Key identification** (illustrated in the following slides):

$$E[Y_1(0) \mid D = 1] = E[Y_0 \mid D = 1] + E[Y_1 - Y_0 \mid D = 0]$$

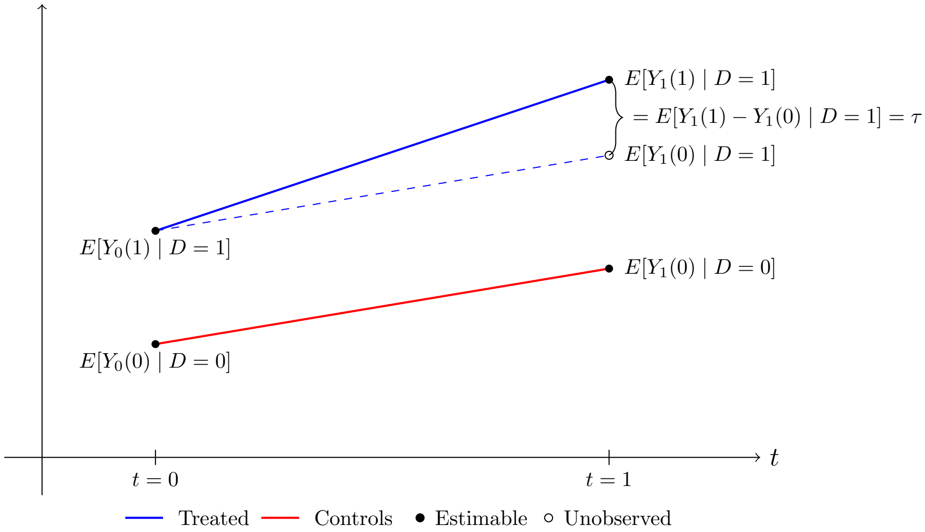
yielding the difference-in-differences:

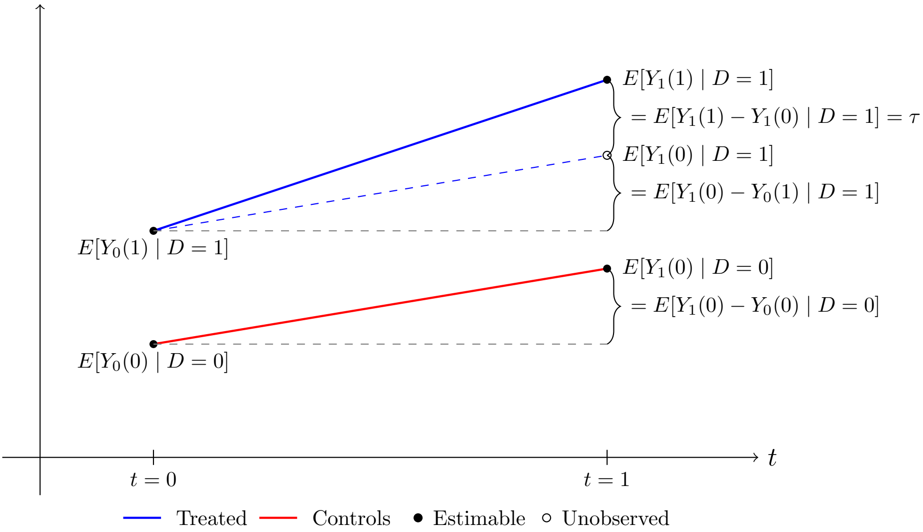
$$\begin{aligned} \tau &= E[Y_1 \mid D = 1] - E[Y_1(0) \mid D = 1] \\ &= E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0 \mid D = 0] \end{aligned}$$


$E[Y_1(1) \mid D = 1]$
$E[Y_0(1) \mid D = 1]$
$E[Y_1(0) \mid D = 0]$
$E[Y_0(0) \mid D = 0]$
$t$
$t = 0$
$t = 1$
Treated
Controls
Estimable



$E[Y_1(1) \mid D = 1]$
$E[Y_1(0) \mid D = 1]$
$E[Y_0(1) \mid D = 1]$
$E[Y_1(0) \mid D = 0]$
$E[Y_0(0) \mid D = 0]$
$t$
$t = 0$
$t = 1$
Treated
Controls
Estimable
Unobserved


$E[Y_1(1) \mid D = 1]$
$= E[Y_1(1) - Y_1(0) \mid D = 1] = \tau$
$E[Y_1(0) \mid D = 1]$
$E[Y_0(1) \mid D = 1]$
$E[Y_1(0) \mid D = 0]$
$E[Y_0(0) \mid D = 0]$
$t$
$t = 0$
$t = 1$
Treated
Controls
Estimable
Unobserved



$E[Y_1(1) \mid D = 1]$
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$E[Y_1(0) \mid D = 0]$
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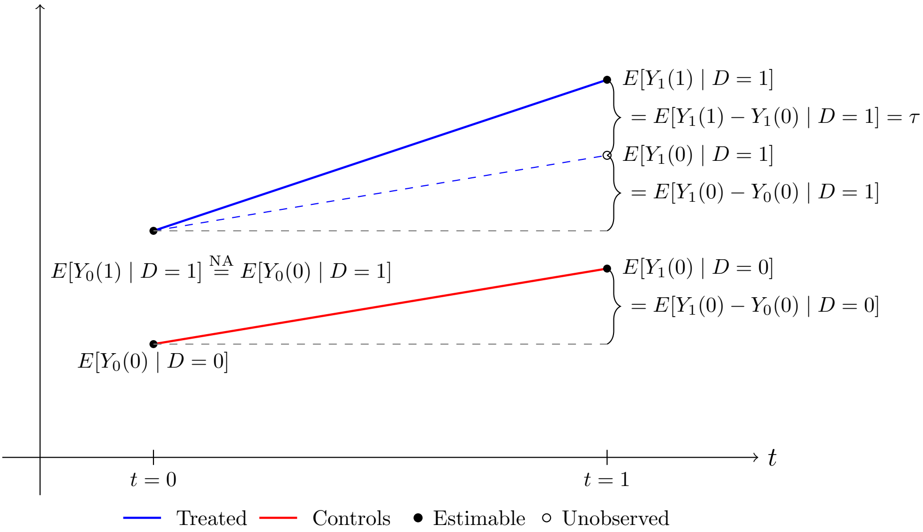
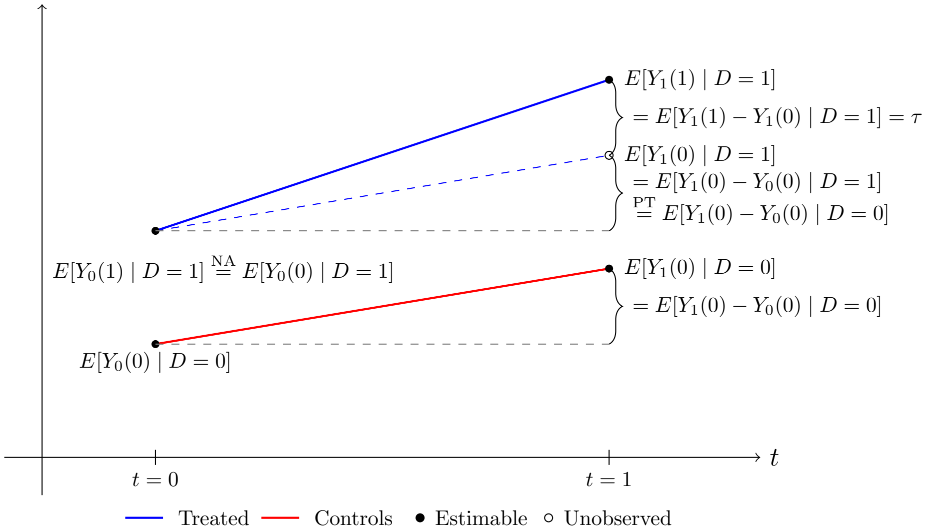
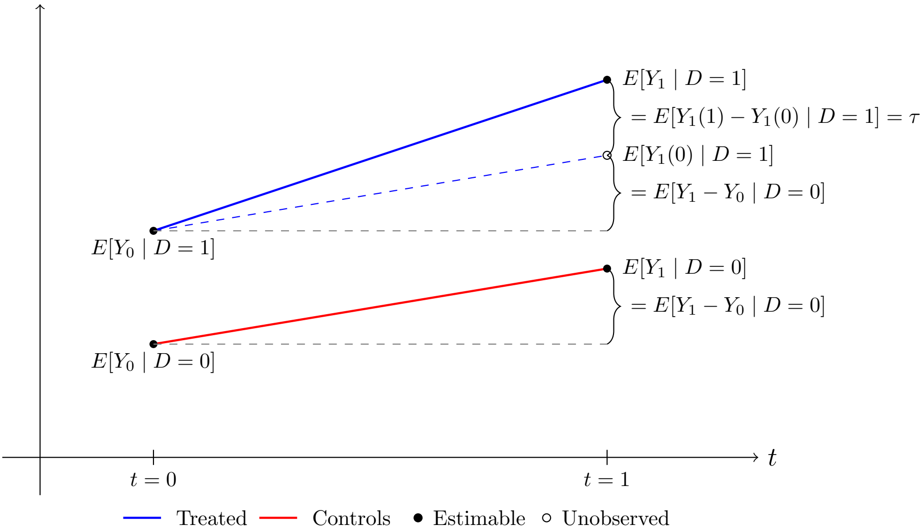
$E[Y_0(0) \mid D = 0]$
$t$
$t = 0$
$t = 1$
Treated
Controls
Estimable
Unobserved



$E[Y_1(1) \mid D=1]$
$= E[Y_1(1) - Y_1(0) \mid D=1] = \tau$
$E[Y_1(0) \mid D=1]$
$= E[Y_1(0) - Y_0(0) \mid D=1]$
$E[Y_0(1) \mid D=1] \overset{\text{NA}}{=} E[Y_0(0) \mid D=1]$
$E[Y_1(0) \mid D=0]$
$= E[Y_1(0) - Y_0(0) \mid D=0]$
$E[Y_0(0) \mid D=0]$
$t$
$t=0$
$t=1$
Treated
Controls
Estimable
Unobserved



$E[Y_1(1) \mid D = 1]$
$= E[Y_1(1) - Y_1(0) \mid D = 1] = \tau$
$E[Y_1(0) \mid D = 1]$
$= E[Y_1(0) - Y_0(0) \mid D = 1]$
$\overset{\text{PT}}{=} E[Y_1(0) - Y_0(0) \mid D = 0]$
$E[Y_0(1) \mid D = 1] \overset{\text{NA}}{=} E[Y_0(0) \mid D = 1]$
$E[Y_1(0) \mid D = 0]$
$= E[Y_1(0) - Y_0(0) \mid D = 0]$
$E[Y_0(0) \mid D = 0]$
$t$
$t = 0$
$t = 1$
Treated
Controls
Estimable
Unobserved



$E[Y_1 \mid D=1]$
$= E[Y_1(1) - Y_1(0) \mid D=1] = \tau$
$E[Y_1(0) \mid D=1]$
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$= E[Y_1 - Y_0 \mid D=0]$
$E[Y_0 \mid D=0]$
$t$
$t=0$
$t=1$
Treated
Controls
Estimable
Unobserved

# Addendum

- The previous slide followed by consistency
- The identification of the key counterfactual, $E[Y_1(0) \mid D = 1]$, is now evident: We take the average outcome of the treated group in the pre-treatment period, $E[Y_0 \mid D = 1]$, and add to it the average trend of the non-treated group, $E[Y_1 - Y_0 \mid D = 0]$
- Inserting the identified counterfactual in the definition of the ATT estimand, $\tau$, yields the estimable difference-in-differences estimand:

$$\begin{aligned} \tau &= E[Y_1 \mid D = 1] - E[Y_1(0) \mid D = 1] \\ &= E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0 \mid D = 0] \end{aligned}$$